Role of initial system-bath correlation on coherence trapping

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We study the coherence trapping of a qubit correlated initially with a non-Markovian bath in a pure dephasing channel. By considering the initial qubit-bath correlation and the bath spectral density, we find that the initial qubit-bath correlation can lead to a more efficient coherence trapping than that of the initially separable qubit-bath state. The stationary coherence in the long time limit can be maximized by optimizing the parameters of the initially correlated qubit-bath state and the bath spectral density. In addition, the effects of this initial correlation on the maximal evolution speed for the qubit trapped to its stationary coherence state are also explored.

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Introduction. Quantum state takes the form of superposition which leads to quantum coherence. Quantum coherence plays a central role in the applications of quantum physics and quantum information science [1, 2]. However, it is fragile due to interactions of the environment. Understanding of quantum coherence dynamics of an open system is a very important task in many areas of physics ranging from quantum optics to quantum information processing. It is known that many quantum open systems exhibit non-Markovian behavior with a flow of information from the environment back to the system [3–7]. This presence of non-Markovian effects can induce the long-lasting coherence in biological surroundings and the steady state entanglement in coherently coupled dimer systems [8, 9]. By considering the pure dephasing non-Markovian bath, decay of quantum coherence of the system would be terminated in a finite time, such that the system can partly retain coherence in the long time limit. This new phenomenon, known as *coherence trapping* [10], is important for quantum information processing since the effective long-time quantum coherence of the system is preserved. Coherence trapping of a quantum system is mainly related to the open dynamics, and is generally analyzed in the fact that the system and bath are initially separable. As is well known, however, the initial system-bath correlations are important for the the dynamics of the open systems. The distinguishability of quantum states would increase in the presence of initial system-bath correlations [17, 18]. The information flow between the system and its bath and the corresponding degree of non-Markovianity can also be influenced by these initial correlations [19–22]. On the other hand, the standard master equation approach to open systems may not be appropriate unless a product state is explicitly prepared [11–16]. So the coherence trapping of an open system due to the initial system-bath correlations should be studied both physically and methodologically.

In this paper, we will concentrate on the following questions: how do the initial system-bath correlations affect coherence trapping of the system? which form of the initially correlated system-bath state can maximize the stationary coherence of the system? We consider the pure dephasing model of a qubit initially correlated with a zero-temperature Ohmic-Like bath. We will show that the initial qubit-bath correlation can lead to the more efficient coherence trapping, while the lower initial coherence of the qubit is induced by this initial correlation. In the long time limit, the stationary coherence of the qubit can be maximized by choosing the optimal parameters of the initially correlated qubit-bath state and the optimal Ohmicity parameter of the bath.

Furthermore, the task to drive an initial state to a prescribed target state in the shortest possible time is significant for quantum control in many areas of physics, such as quantum computation [23], fast population transfer in quantum optics [24], and quantum optimal control protocols [25, 26]. This minimum evolution time, which is defined as quantum speed limit (QSL) time [27–40], is a key method in characterizing the maximal speed of evolution of quantum systems. Here in order to speed up the evolution from an initial coherence state to its stationary coherence state, we further focus on the interactions of the initial qubit-bath correlated state, the spectral density function of the bath and the QSL time. Remarkably, we find that the initial qubit-bath correlation can reduce the QSL time for the occurrence of coherence trapping. The maximal evolution speed for the qubit trapped to its stationary coherence state can also be controlled by optimizing the parameters of the initial qubit-bath correlated state and the bath spectral density function.

Model. Let us consider an exactly solvable model, in which the process of energy dissipation is negligible and only pure depahsing is a mechanism for decoherence of

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the qubit. The associated Hamiltonian reads (setting $\hbar = 1$),

$$H = \omega_0\sigma_z + \int_0^\infty \omega a_\omega^\dagger a_\omega d\omega + \int_0^\infty \sigma_z[g_\omega a_\omega^\dagger + g_\omega^* a_\omega]d\omega, \quad (1)$$

where the operator σ_z is defined by $\sigma_z = |e\rangle\langle e| - |g\rangle\langle g|$, associated with the upper level $|e\rangle$ and the lower level $|g\rangle$ of the qubit; a_ω and $a_\omega^\dagger$ are the bosonic annihilation and creation operators for the bath, which is characterized by the frequency ω; g_ω is the coupling constant of the interactions of the qubit with the bath, and g_ω^* is the complex conjugate to g_ω. The Hamiltonian in Eq. (1) can be rewritten in the block-diagonal structure [41, 42] $H = diag[H_e, H_g]$, where $H_{e/g} = \pm\omega_0 + \int_0^\infty \omega a_\omega^\dagger a_\omega d\omega \pm \int_0^\infty [g_\omega a_\omega^\dagger + g_\omega^* a_\omega]d\omega$.

Here, we consider the situation where a correlated initial state of the qubit-bath system in the form [18],

$$|\Psi(0)\rangle = c_e|e\rangle \otimes |\xi_0\rangle + c_g|g\rangle \otimes |\xi_\lambda\rangle, \quad (2)$$

with the non-zero complex numbers $c_{g/e}$ are satisfied $|c_e|^2 + |c_g|^2 = 1$. And we assume that $|\xi_0\rangle$ is a bath ground state and $|\xi_\lambda\rangle = C_\lambda^{-1}[(1-\lambda)|\xi_0\rangle + \lambda|\xi_f\rangle]$ is a bath superposition state of the ground state $|\xi_0\rangle$ and a coherent state $|\xi_f\rangle = D(f)|\xi_0\rangle$. The displacement operator $D(f)$ reads $D(f) = \exp\{\int_0^\infty [f_\omega a_\omega^\dagger - f_\omega^* a_\omega]d\omega\}$ for an arbitrary square-integrable function f. The constant $C_\lambda = \sqrt{(1-\lambda)^2 + \lambda^2 + 2\lambda(1-\lambda)Re\langle\xi_0|\xi_f\rangle}$ normalizes the state $|\xi_\lambda\rangle$, where Re is a real part of $\langle\xi_0|\xi_f\rangle$ in the bath Hilbert space. The correlation parameter $\lambda \in [0, 1]$ determines the initial correlation of the qubit and bath. Through performing the Hamiltonian described in Eq. (1), the state of the total system at any time t is given by $|\Psi(t)\rangle = c_e|e\rangle \otimes |\psi_e(t)\rangle + c_g|g\rangle \otimes |\psi_g(t)\rangle$, where $|\psi_e(t)\rangle = exp(-iH_e t)|\xi_0\rangle$ and $|\psi_g(t)\rangle = exp(-iH_g t)|\xi_\lambda\rangle$. Then the reduced density matrix $\rho_\lambda(t)$ of the qubit at time t reads, $\rho_{ee}(t) = |c_e|^2$, $\rho_{gg}(t) = |c_g|^2$ and $\rho_{eg}(t) = \rho_{ge}^*(t) = c_e c_g^* \Upsilon_\lambda(t)$, with the dephasing rate $\Upsilon_\lambda(t)$.

The qubit dynamics is closely dictated by the spectral density function characterising the qubit-bath interaction. In the following the bath can be described by the family of Ohmic-Like spectra $|g_\omega|^2 = \alpha\omega^{\mu+1}\exp(-\omega/\omega_c)$, with ω_c being the cutoff frequency and $\alpha > 0$ a dimensionless coupling constant. By changing the μ-parameter, one goes from sub-Ohmic baths ($-1 < \mu < 0$) to Ohmic ($\mu = 0$) and super-Ohmic ($\mu > 0$) baths, respectively. Furthermore, the coherent state $|\xi_f\rangle$ can be calculated by the spectral density function $|f_\omega|^2 = \omega^{\upsilon+1}\exp(-\omega/\omega_c)$, with $\upsilon > 0$. So the initial state of the qubit-bath system can be controlled by the parameters λ and υ. For the case $\lambda = 0$ the qubit and the bath are initially uncorrelated, the dephasing rate can be obtained, $\Upsilon_0(t) = \exp[-2i\omega_0 t - r(t)]$. While for $0 < \lambda \leq 1$ the initial correlation exists in the qubit-bath system, one also finds, $\Upsilon_\lambda(t) = C_\lambda^{-1}\{1-\lambda+\lambda\exp[-2i\phi(t)+k(t)]\}\exp[-2i\omega_0 t - r(t)]$, where,

$$\begin{aligned} r(t) &= 4\alpha\Gamma[\mu]\omega_c^\mu\{1 - \frac{\cos[\mu\arctan(\omega_c t)]}{(1+\omega_c^2 t^2)^{\mu/2}}\}, \\ k(t) &= 2\sqrt{\alpha}\Gamma[\vartheta]\omega_c^\vartheta\{1 - \frac{\cos[\vartheta\arctan(\omega_c t)]}{(1+\omega_c^2 t^2)^{\vartheta/2}}\} - \frac{1}{2}\Gamma[\upsilon]\omega_c^\upsilon, \\ \phi(t) &= \sqrt{\alpha}\Gamma[\vartheta]\omega_c^\vartheta\frac{\sin[\vartheta\arctan(\omega_c t)]}{(1+\omega_c^2 t^2)^{\vartheta/2}}, \end{aligned} \quad (3)$$

where $\Gamma[\cdot]$ is the Euler gamma function and the parameter $\vartheta = (\mu+\upsilon)/2$.

Coherence trapping for the qubit. How to quantify quantum coherence of a quantum system now becomes paramountly important. In recent years, a wide variety of measures of coherence have been proposed [43–45]. Currently, Baumgratz, Cramer and Plenio find that the relative entropy of coherence [43],

$$C(\rho) = S(\rho_{diag}) - S(\rho), \quad (4)$$

where $S(\rho)$ is the von Neumann entropy and ρ_{diag} denotes the state obtained from ρ by deleting all off-diagonal elements, and the intuitive l_1 norm of coherence,

$$C_{l_1}(\rho) = \sum_{i,j,i\neq j} |\rho_{ij}|, \quad (5)$$

are both general and proper measures of coherence. So in the following we will choose the relative entropy of coherence $C(\rho)$ to measure the quantum coherence of the reduced density matrix $\rho_\lambda(t)$ of the qubit in the presence of qubit-bath initial correlation.

If there is no correlations in the initial qubit-bath state, the qubit dephasing $\Upsilon_0(t)$ is characterized by exponential decay of the qubit coherence, hence will predict vanishing coherence in the long time limit in the Ohmic and sub-Ohmic dephasing baths [10]. While for the super-Ohmic baths, the qubit dephasing will stop after a finite time, therefore leading to coherence trapping. This behavior can realize the effective long-time coherence protection. In the following, we would mainly see the effect of the initially correlated qubit-bath state on coherence trapping of a qubit in the super-Ohmic bath model. The preparation of this initially correlated qubit-bath state can be obtained by non-local operations with two steps [18]. Firstly, one prepares the bath state $|\xi_\lambda\rangle$, and then products it with the ground state of the qubit $|g\rangle$. Then, one would superpose $|g\rangle \otimes |\xi_\lambda\rangle$ with the product state $|e\rangle \otimes |\xi_0\rangle$, with the weights $c_{g/e}$, respectively. And the initial correlations of the qubit-bath system can be controlled by the parameters $c_{g/e}$, λ and the function f_ω.

We shall examine the decoherence process where the initially correlated qubit-bath state is in the form of Eq. (2), with $c_e = c_g = 1/\sqrt{2}$. Then the initial coherence of the qubit can be evaluated $C(\rho_\lambda^{t=0}) = \frac{1}{2}(1 - \Upsilon_\lambda(0))\log_2[1-\Upsilon_\lambda(0)] + \frac{1}{2}(1+\Upsilon_\lambda(0))\log_2[1+\Upsilon_\lambda(0)]$, with $\Upsilon_\lambda(0) = C_\lambda^{-1}(1-\lambda+\lambda\exp[-\frac{1}{2}\Gamma[\upsilon]\omega_c^\upsilon])$. At time $t = 0$,

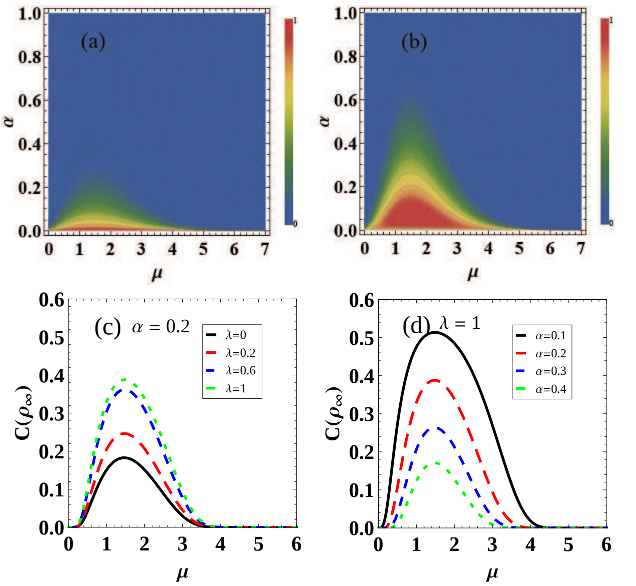

FIG. 1: (Color online) The stationary coherence of the qubit quantified by the relative entropy of coherence $C(\rho_\infty)$ as a function of the bath parameters α and μ. (a) for the uncorrelated initial qubit-bath state ($\lambda = 0$); (b) for the correlated initial qubit-bath state ($\lambda = 1$); (c) $\alpha = 0.2$; (d) $\lambda = 1$. Parameters are chosen as, $\upsilon = 1.5$, and $\omega_c = 1$.

in the case $\lambda = 0$ the dephasing rate $\Upsilon_0(0) = 1$, while for the correlated initial state we can obtain $0 < \Upsilon_\lambda(0) < 1$. So it means $C(\rho_{t=0}^{\lambda\neq0}) < C(\rho_{t=0}^{\lambda=0})$, that is to say the initial correlation of the qubit-bath system can lead to lower initial coherence of the qubit.

On the other hand, to clear the effect of the qubit-bath initial correlation explicitly, we also perform the calculation for the stationary value of coherence trapping in the long time limit. In Fig. 1, we show the stationary coherence $C(\rho_\infty)$ between the initially uncorrelated $\lambda = 0$ and correlated $\lambda = 1$ states as a function of the bath parameters α and μ. By comparing Figs. 1(a) and 1(b), it is clear that the presence of the qubit-bath correlation in the initial state enlarges the region for occurrence of coherence trapping. Moreover, by giving the other parameters, Fig. 1(c) clearly shows that the larger correlation parameter λ leads to a more efficient coherence trapping as the stationary coherence is higher than that obtained from the initially uncorrelated qubit-bath state. Although the lower initial coherence of the qubit can be induced by the correlation parameter λ, the coherences of the bath subsystem and the qubit-bath composite system would appear in the initial qubit-bath state correspondingly. That is the main physical reason of the more efficient coherence trapping of the qubit induced by the correlated initial qubit-bath state. Additionally, from Fig. 1 we also can easily find that, the stronger coupling α of the qubit to bath diminishes the stationary coherence in the long time limit. And there exists an optimal value of the Ohmicity parameter $\mu \doteq 1.46$ of the bath maximizing the stationary coherence, which is independent of the coupling constant α and the correlation parameter λ, as shown in Figs. 1(c) and 1(d).

Next, by choosing the optimal value $\mu = 1.46$ of the super-Ohmic bath, the influence of the parameters characterizing the initially correlated state on coherence trapping is depicted in Fig. 2(a). Two regions, the enhancing of coherence tapping (ECT) (i.e. $C(\rho_\infty^{\lambda\neq0}) > C(\rho_\infty^{\lambda=0}) = 0.1827$) and the no-enhancing of coherence trapping (No-ECT) (i.e. $C(\rho_\infty^{\lambda\neq0}) \leq C(\rho_\infty^{\lambda=0})$), are acquired in the corresponding parameter planes. The dashed-white line $C(\rho_\infty) = 0.1827$ is the dividing line between these two regions. That is to say, not all but specific initial states $|\xi_\lambda\rangle$ can lead to the enhancing coherence trapping. The range of υ to gain the enhancing of coherence trapping, would reduce as the correlation parameter λ increasing, as shown in Fig. 2(b). So we conclude that, in order to achieve the most efficient coherence in the long time limit, both the optimal Ohmicity parameter μ and the optimal state $|\xi_\lambda\rangle$ must be satisfied.

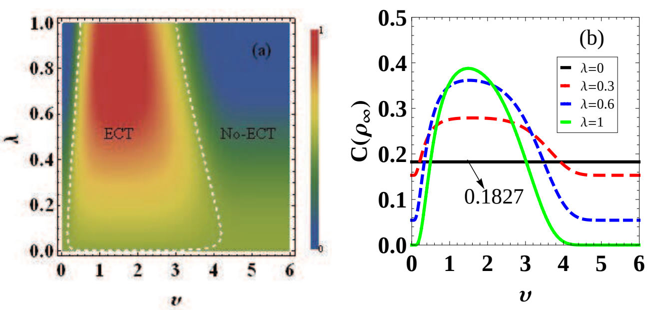

FIG. 2: (Color online) The stationary coherence of the qubit quantified by the relative entropy of coherence $C(\rho_\infty)$ as a function of the parameters for the initial qubit-bath state λ and υ. The dashed-white line in (a) means $C(\rho_\infty) = 0.1827$, which is the dividing line between two regions. Parameters are chosen as, $\alpha = 0.2$, $\mu = 1.46$, and $\omega_c = 1$.

Quantum evolution speed. Since the qubit would occur coherence trapping after a finite time t_c in the super-Ohmic bath, one may naturally concern the evolution speed between the initial state $\rho_\lambda(0)$ and the stationary coherence state $\rho_\lambda(t_c)$. The quantum speed of evolution from $\rho_\lambda(0)$ to its target state $\rho_\lambda(t_c)$ can be characterized by QSL time [38, 39]. The definition of QSL time between an arbitrary initially mixed state ρ_0 and its target state ρ_τ, governed by the master equation $\dot{\rho}_t = L_t\rho_t$, with L_t the positive generator of the dynamical semigroup, is as follows [39] $\tau_{QSL} = \max\{\frac{1}{\overline{\sum_{i=1}^n \sigma_i \varrho_i}}, \frac{1}{\overline{\sqrt{\sum_{i=1}^n \sigma_i^2}}}\}B(\rho_0, \rho_\tau)$, with $\overline{X} = \tau^{-1}\int_0^\tau X dt$, $B(\rho_0, \rho_\tau) = |tr(\rho_0\rho_\tau) - tr(\rho_0^2)|$ denotes a metric on the space of the initial state ρ_0 and the target state ρ_τ via the so-called relative purity, and σ_i are the singular values of $\dot{\rho}_t$ and ϱ_i those of the initial mixed state ρ_0. The above expression of τ_{QSL} can effectually define the minimal evolution time for arbitrary initial states, and also be used to assess quantum evolution speed of open quantum system.

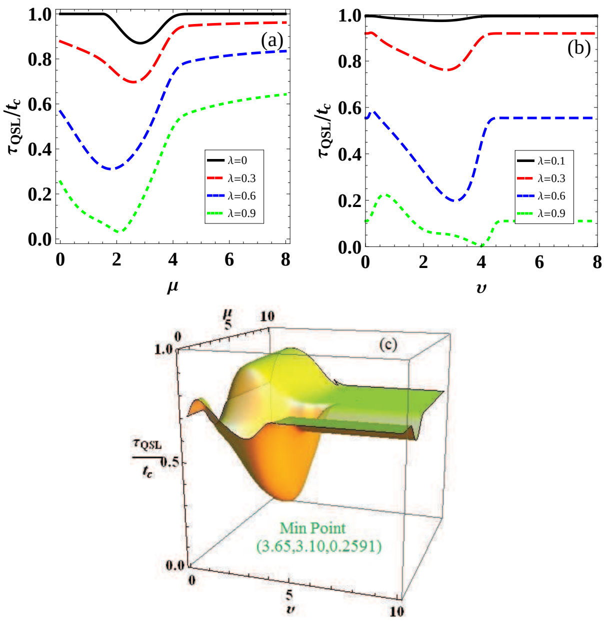

FIG. 3: (Color online) The QSL time for the evolution from the initial state $\rho_\lambda(0)$ to the stationary coherence state $\rho_\lambda(t_c)$, quantified by τ_{QSL}/t_c as a function of the parameters μ and υ, with $\omega_c = 1$. Parameters are chosen as, (a) $\alpha = 0.2$, $\upsilon = 2$; (b) $\alpha = 0.2$, $\mu = 1.46$; (c) $\alpha = 0.2$, $\lambda = 0.5$.

Here, we also consider the weights $c_e = c_g = 1/\sqrt{2}$ in the initially correlated qubit-bath state in Eq. (2). Then the QSL time for the qubit initial state $\rho_\lambda(0)$ to the stationary coherence state $\rho_\lambda(t_c)$, can be calculated $\tau_{QSL}/t_c = |\Upsilon_\lambda(0)[\Upsilon_\lambda(t_c) - \Upsilon_\lambda(0)]|/\int_0^{t_c} |\dot{\Upsilon}_\lambda(t)|dt$. In Figs. $3(a)$ and $3(b)$, we demonstrate how the QSL time for evolution from $\rho_\lambda(0)$ to $\rho_\lambda(t_c)$ can depend on the parameters μ and υ, with different selected correlation parameter λ. Firstly, it is clear that the initial qubit-bath correlation can reduce the QSL time as the value of λ increasing. That is to say, the evolution from the initial coherence state to the stationary coherence state, can be speeded up by the initial correlation in the qubit-bath state. And then, another remarkable feature can be acquired: There exist the optimal Ohmicity parameter μ or the parameter υ of $|\xi_f\rangle$, which can induce the minimum value of QSL time. And the optimal parameters μ or υ are dependent of the correlation parameter λ. In Fig. $3(a)$, when $\upsilon = 2$, the optimal Ohmicity parameter $\mu \doteq 2.84, 2.60, 1.80, 2.09$ for $\lambda = 0, 0.3, 0.6, 0.9$, respectively. By choosing $\mu = 1.46$, as shown in Fig. $3(b)$, the optimal parameter for the initial bath state $|\xi_f\rangle$ can be obtain $\upsilon \doteq 2.61, 2.80, 3.09, 3.97$ for $\lambda = 0.1, 0.3, 0.6, 0.9$, respectively.

Furthermore, since both the Ohmicity parameter μ and the parameter υ of $|\xi_f\rangle$ can bring about the minimum QSL time, in the following we would seek the optimal condition (υ, μ) on the maximal evolution speed of the qubit. Fig. $3(c)$ shows QSL time for $\rho_\lambda(0)$ to $\rho_\lambda(t_c)$ as a function of μ and υ. By a given correlation parameter $\lambda = 0.5$, we observe that, the minimum QSL time can only appear in the region $(\upsilon < 5, \mu < 4)$. And the optimal values $(\upsilon = 3.65, \mu = 3.10)$ which lead to the minimum QSL time $\tau_{QSL}^{min}/t_c = 0.2591$, can be found by accurate numerical calculation. This can be understand that, in order to speed up the evolution speed of the qubit, the Ohmicity parameter μ and the parameter υ of $|\xi_f\rangle$ should be optimized. Combined with the above section about coherence trapping, the aim to make the qubit trap in a higher stationary coherence state with the maximal evolution speed, can be attained by choosing the optimal parameters of the initial qubit-bath state (λ, υ) and the bath spectral density function (μ).

Conclusion. In summary, we studied intriguing features of coherence trapping of a qubit with a zero-temperature structured bath by considering the initial qubit-bath correlation. The initial qubit-bath correlation not only leads to a more efficient coherence trapping, but also speeds up the evolution for the occurrence of coherence trapping. Moreover, both the maximum stationary coherence in the long time limit and the minimum QSL time from the initial state to the stationary coherence state, can be acquired by optimizing the parameters of the initially correlated qubit-bath state and the bath spectral density. This physical mechanism leading quickly to a higher stationary coherence would play an important role for implementing quantum simulators [46] and quantum information processors [47]. It is worth pointing out that the non-Markovian effects may not monotonically cause the acceleration of the system evolution in the super-Ohmic bath, as shown in Fig. $3(a)$. This is clearly different from the main result in the damped Jaynes-Cummings model [38], which shows that the evolution speed can be monotonically increased by non-Markovian effects. So the specific interplay between the evolution speed of the system and the bath non-Markovian effects should be studied under different circumstances. Experimentally, the coherence trapping can be demonstrated by qubit-bath systems like optics [21], trapped ions [48] and superconducting qubit [47, 49, 50].

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